Pearson Edexcel Level 3 GCE

Centre Number

Candidate Number

Monday 19 October 2020

Afternoon   Paper Reference **9MA0/31**

Mathematics

Advanced

Paper 31: Statistics   **Shadow Paper Set 1**

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations.
Calculators must not have the facility for symbolic algebra manipulation,
differentiation and integration, or have retrievable mathematical formulae
stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name,
  centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are
  clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear.
  Answers without working may not gain full credit.
- Values from statistical tables should be quoted in full. If a calculator is used instead
  of tables the value should be given to an equivalent degree of accuracy.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 50. There are 5 questions.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

1   The Venn diagram shows the probabilities associated with four events, *P*, *Q*, *R* and *S*
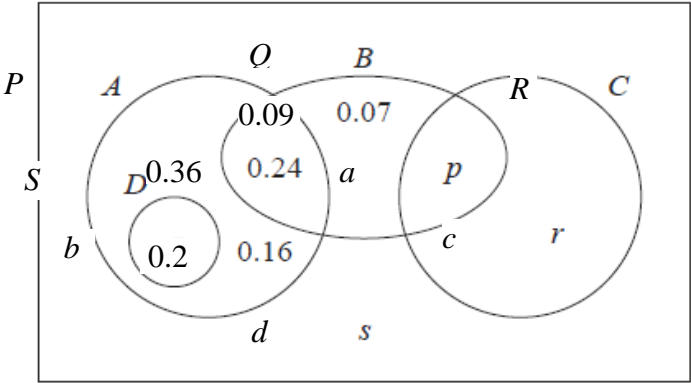
(*a*) Write down any pair of mutually exclusive events from *P*, *Q*, *R* and *S*

(1)

Given that P(*Q*) = 0.5

(*b*) find the value of *a*

(1)

Given also that *P* and *Q* are independent

(*c*) find the value of *b*

(2)

Given further that P(*Q′* | *R*) = 0.6

(*d*) find

(i)   the value of *c*

(ii) the value of *d*

(4)

(Total for Question 1 is 8 marks)

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2 A random sample of 15 days is taken from the large data set for Perth in June and July 1987.
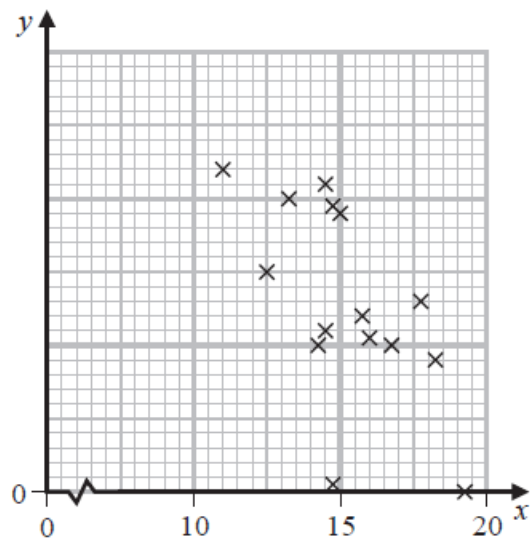The scatter diagram in Figure 1 displays the values of two of the variables for these 15 days.

Figure 1

(*a*) Describe the correlation.

(1)

The variable on the *x*-axis is Daily Mean Temperature measured in °C.

(*b*) Using your knowledge of the large data set,

   (i) suggest which variable is on the *y*-axis,

   (ii) state the units that are used in the large data set for this variable.

(2)

Priti believes that there is a correlation between Daily Total Sunshine and Daily Maximum Relative Humidity at Heathrow.

She calculates the product moment correlation coefficient between these two variables for a random sample of 25 days and obtains $r = -0.377$

(*c*) Carry out a suitable test to investigate Priti's belief at a 5% level of significance.
State clearly

- your hypotheses
- your critical value

(3)

On a random day at Heathrow the Daily Maximum Relative Humidity was 95%

(*d*) Comment on the number of hours of sunshine you would expect on that day, giving a reason for your answer.

(1)

(Total for Question 2 is 7 marks)

3 Each member of a group of 35 people went walking on a weekend.

The time taken for the walk, x minutes, for each member of the group was recorded.

These times are summarised in the following box and whisker plot.

							70
0	60	120	180	240	300	360	420

Time (x minutes) inutes)
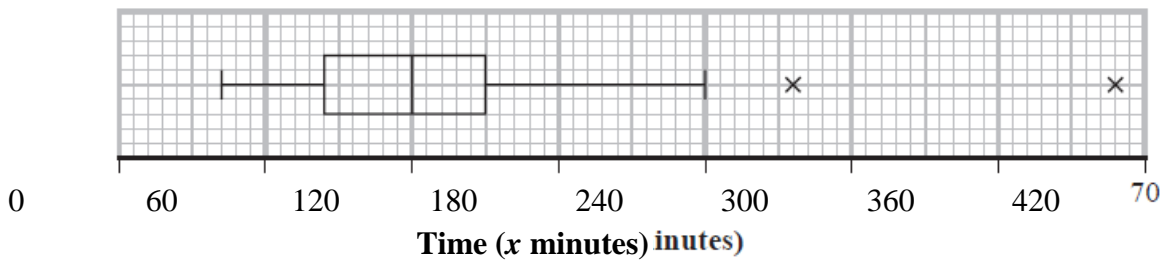
(*a*) Find the range of the times.

(1)

(*b*) Find the interquartile range of the times.

(1)

For these 35 people $\sum x = 3850$ and $\sum x^2 = 575960$

(*c*) calculate the mean time taken on the walk,

(1)

(*d*) calculate the standard deviation of the times taken on the walk.

(2)

Louise defines an outlier as a value more than 3 standard deviations above the mean.

(*e*) State how many outliers Louise would say there are in these data, giving a reason for your answer.

(1)

Alana and Buda also went walking for a minutes and b minutes respectively, where $a > b$.

When their times are included with the data of the other 35 people

- the median time decreases
- the mean time does not change

(*f*) Suggest a possible value for a and a possible value for b, explaining how your values satisfy the above conditions.

(3)

(*g*) Without carrying out any further calculations, explain why the standard deviation of all 37 times will be lower than your answer to part (*d*).

(1)

(Total for Question 3 is 10 marks)

4 The discrete random variable D has the following probability distribution

d	5	10	15	20	25
P($D = d$)	$\dfrac{c}{5}$	$\dfrac{c}{10}$	$\dfrac{c}{15}$	$\dfrac{c}{20}$	$\dfrac{c}{25}$

where c is a constant.

(a) Show that the value of c is $\dfrac{300}{137}$

(2)

The random variables D_1 and D_2 are independent and each have the same distribution as D.

(b) Find P $(D_1 + D_2 = 40)$.
    Give your answer to 3 significant figures.

(3)

A single observation of D is made.

The value obtained, d, is the common difference of an arithmetic sequence.

The first 4 terms of this arithmetic sequence are the angles, measured in degrees, of quadrilateral Q

(c) Find the exact probability that the smallest angle of Q is more than 65°

(5)

(Total for Question 4 is 10 marks)

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5 A health centre claims that the time a dentist spends with a patient can be modelled by a normal distribution with a mean of 25 minutes and a standard deviation of 12 minutes.

(*a*) Using this model, find the probability that the time spent with a randomly selected patient is more than 40 minutes.

(1)

Some patients complain that the mean time the dentist spends with a patient is more than 25 minutes.

The receptionist takes a random sample of 30 patients and finds that the mean time the doctor spends with a patient is 28 minutes.

(*b*) Stating your hypotheses clearly and using a 5% significance level, test whether or not there is evidence to support the patients' complaint.

(4)

The health centre also claims that the time a receptionist spends registering a patient for an appointment, T minutes, can be modelled by the normal distribution where $T \sim N(4, 2.5^2)$

(*c*) Using this model,

    (i)   find the probability that a registration with the receptionist takes less than 3 minutes,

(1)

    (ii) find P $(T < 3 \mid T > 0)$

(3)

    (iii) hence explain why this normal distribution may not be a good model for T.

(1)

The receptionist believes that she cannot complete registration in less than 3 minutes.

She suggests that the dental centre should use a refined model only including values of $T > 3$.

(*d*) Find the median time to be registered using this new model, giving your answer correct to one decimal place.

(5)

(Total for Question 5 is 15 marks)

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TOTAL FOR STATISTICS IS 50 MARKS

Candidate surname

Other names

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Mathematics

Advanced
Paper 32: Mechanics

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Total Marks

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- Answer the questions in the spaces provided
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- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Unless otherwise indicated, whenever a value of g is required, take $g = 9.8$ m s^{-2} and give your answer to either 2 significant figures or 3 significant figures.

Information

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Advice

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- Try to answer every question.
- Check your answers if you have time at the end.

1   A rough plane is inclined to the horizontal at an angle α, where $\tan \alpha = \dfrac{5}{12}$

A brick B of mass m is placed on the plane.

The coefficient of friction between B and the plane is μ

Brick B is in equilibrium and on the point of sliding down the plane.

Brick B is modelled as a particle.

Using the model,

(*a*) find, in terms of m and g, the magnitude of the normal reaction of the plane on brick B

(2)

(*b*) show that $\mu = \dfrac{5}{12}$

(4)

For parts (c) and (d), you are not required to do any further calculations.

Brick B is now removed from the plane and a much heavier brick C is placed on the plane.
The coefficient of friction between C and the plane is also $\dfrac{5}{12}$

(*c*) Explain briefly why brick C will remain at rest on the plane.

(1)

Brick C is now projected with speed 0.6 m s^{-1} down a line of greatest slope of the plane.

Brick C is modelled as a particle.

Using the model,

(*d*) describe the motion of brick C, giving a reason for your answer.

(2)

(Total for Question 1 is 9 marks)

2 A particle P moves with acceleration $(5\mathbf{i} - 4\mathbf{j})$ m s^{-2}

At time $t = 0$, P is moving with velocity $(-2\mathbf{i} + 3\mathbf{j})$ m s^{-1}

(a) Find the velocity of P at time $t = 3$ seconds.

(2)

At time $t = 0$, P passes through the origin O.

At time $t = T$ seconds, where $T > 0$, the particle P passes through the point A.

The position vector of A is $(\lambda\mathbf{i} - 5\mathbf{j})$m relative to O, where λ is a constant.

(b) Find the value of T.

(4)

(c) Hence find the value of λ

(2)

(Total for Question 2 is 8 marks)

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3 (i) At time t seconds, where $t \geq 0$, a particle P moves so that its acceleration $\mathbf{a}$ m s^{-2} is given by

$$\mathbf{a} = (2 - 8t)\,\mathbf{i} + (5 - t^2)\,\mathbf{j}$$

At the instant when $t = 0$, the velocity of P is $156\mathbf{i}$ m s^{-1}

(a) Find the velocity of P when $t = 5$

(3)

(b) Find the value of t at the instant when P is moving in a direction perpendicular to $\mathbf{i}$

(3)

(ii) At time t seconds, where $t \geq 0$, a particle Q moves so that its position vector $\mathbf{r}$ metres, relative to a fixed origin O, is given by

$$\mathbf{r} = (2t^2 - 2t)\,\mathbf{i} + 5t\,\mathbf{j}$$

Find the value of t at the instant when the speed of Q is 13 m s^{-1}

(6)

(Total for Question 3 is 12 marks)

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4

Figure 1
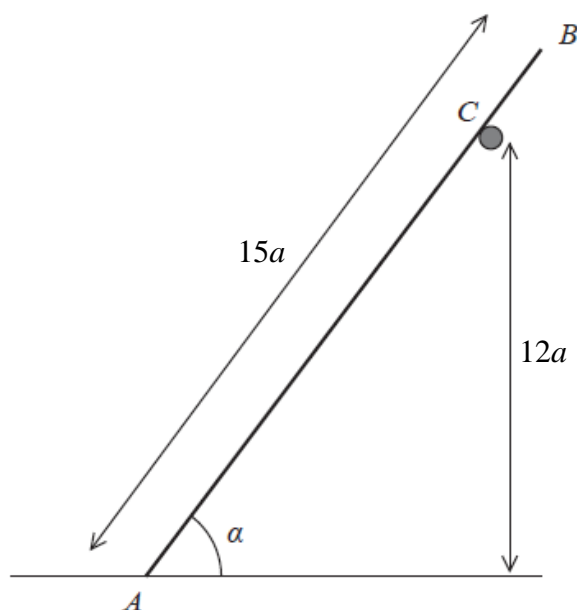
A ladder AB has mass M and length $15a$.

The end A of the ladder is on rough horizontal ground.

The ladder rests against a fixed smooth horizontal rail at the point C.

The point C is at a vertical height $12a$ above the ground.

The vertical plane containing AB is perpendicular to the rail.

The ladder is inclined to the horizontal at an angle α, where $\sin \alpha = \dfrac{12}{13}$ as shown in Figure 1.

The coefficient of friction between the ladder and the ground is μ.

The ladder rests in limiting equilibrium.

The ladder is modelled as a uniform rod.

Using the model,

(a) show that the magnitude of the force exerted on the ladder by the rail at C is $\dfrac{75Mg}{338}$

(3)

(b) Hence, or otherwise, find the value of μ.

(7)

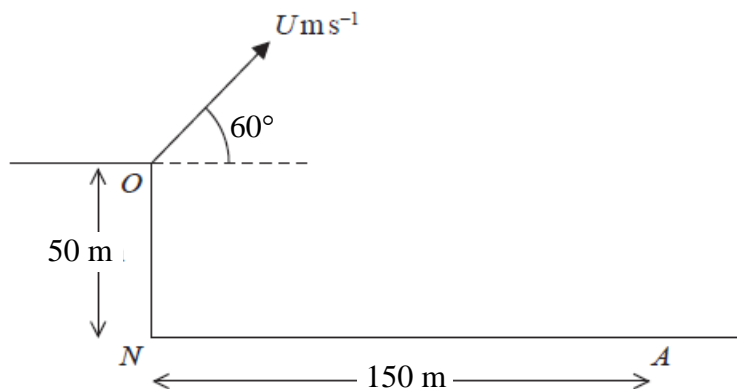**(Total for Question 4 is 10 marks)**

5

Figure 2

A small ball is projected with speed U m s^{-1} from a point O at the top of a vertical cliff.

The point O is 50 m vertically above the point N which is on horizontal ground.

The ball is projected at an angle of 60° above the horizontal.

The ball hits the ground at a point A, where $AN = 150$ m, as shown in Figure 2.

The motion of the ball is modelled as that of a particle moving freely under gravity.

Using this initial model,

(a) show that $U = 37.7$ to 3 significant figures

(6)

(b) taking $U = 37.7$, find the greatest height of the ball above the horizontal ground NA.

(3)

In a refinement to the model of the motion of the ball from O to A, the effect of air resistance is included.

This refined model is used to find a new value of U.

(c) How would this new value of U compare with 37.7, the value given in part (a)?

(1)

(d) State one further refinement to the model that would make the model more realistic.

(1)

(Total for Question 5 is 11 marks)

TOTAL FOR MECHANICS IS 50 MARKS